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GV and GW invariants via the enhanced movable cone

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Abstract

Given any smooth germ of a threefold flopping contraction, we first give a combinatorial characterisation of which Gopakumar–Vafa (GV) invariants are non-zero, by prescribing multiplicities to the walls in the movable cone. On the Gromov–Witten (GW) side, this allows us to describe, and even draw, the critical locus of the associated quantum potential. We prove that the critical locus is the infinite hyperplane arrangement of Iyama and the second author, and moreover that the quantum potential can be reconstructed from a finite fundamental domain. We then iterate, obtaining a combinatorial description of the matrix which controls the transformation of the non-zero GV invariants under a flop. There are three main ingredients and applications: (1) a construction of flops from simultaneous resolution via cosets, which describes how the dual graph changes; (2) a closed formula which describes the change in dimension of the contraction algebra under flop; and (3) a direct and explicit isomorphism between quantum cohomologies of different crepant resolutions, giving a Coxeter-style, visual proof of the Crepant Transformation Conjecture for isolated compound Du Val (cDV) singularities.

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